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by Richard A. Craig

Prepared under Contract No. NASw-704 *by*
GEOPHYSICS CORPORATION OF AMERICA
Bedford, Mass.
for

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ABSTRACT

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The theory of atmospheric tides, as it has been developed for Earth's atmosphere, is applied to the Martian atmosphere. For corresponding modes of oscillation, equivalent depths are less in the Martian atmosphere than in Earth's atmosphere. On the other hand, the eigenvalue corresponding to the presumed Martian temperature distribution in the troposphere and stratosphere is about 20 km, about twice the corresponding value on Earth. These differences arise mainly from the different radii and masses of the planets. Unless the temperature distribution at high levels on Mars has a rather special form so that a second eigenvalue appears, no resonance magnification is to be expected. Tides in the Martian atmosphere might arise from periodic temperature oscillations, induced either by surface heating, or by radiative heating through deep layers of the atmosphere. If there is no significant resonance, and unless these temperature oscillations are much larger than presently believed, the resulting tidal oscillations are unlikely to play any significant role in the circulation of the Martian atmosphere.

Author

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SECTION 1

INTRODUCTION

The expression atmospheric tides is generally used to refer to atmospheric oscillations whose periods are equal to or sub-multiples of solar or lunar day, regardless of whether the oscillations are gravitationally or thermally excited. The theory of tides in the Earth's atmosphere has a long history and has claimed the attention of such men as Laplace, Kelvin, Rayleigh, Margules, Lamb, Chapman, Taylor, and Pekeris. One reason for this interest in the relatively small oscillations involved is the rather surprising observation that in our atmosphere the solar semi-diurnal tide predominates over the lunar tide (with its stronger gravitational excitation) and the solar diurnal tide (with its stronger thermal excitation). Although all the details of this phenomenon are not clearly understood, it now appears that the predominance of solar tides over lunar is due to the importance of thermal excitation and that the relative strength of the solar semi-diurnal oscillation results from a peculiar response of our atmosphere to the periodic heating. There is no reason to believe that the theory, which necessarily contains many simplifications, is inadequate and one may apply it with reasonable confidence to the atmospheres of other planets. The history and present status of tidal theory has been reviewed recently by Siebert (1961), to whose work the reader is referred for further details.

The planet Mars is one for which the application of tidal theory appears to have some interest. The relatively large diurnal temperature variation that is inferred for the surface of Mars raises the question of the response thereto of the Martian atmosphere. Both Mintz (1961) and Ohring and Coté (1963) have speculated on the possibility of an important diurnal oscillation in the Martian atmosphere. The purpose of this report is to investigate such a possibility with the aid of standard tidal theory.

In full detail, the development of this theory is rather cumbersome. Section 2, therefore, contains only a statement of the assumptions and results. Siebert (1961), whose notation is principally used throughout, has given all necessary details and appropriate references. The theory gives rise to two ordinary differential equations, one of which is usually referred to as Laplace's tidal equation and the other of which we may call the radial equation. Solutions of these equations for different conditions, with reference to both Earth and Mars, are discussed in Section 3. Finally, in Section 4, some numerical estimates for Mars are presented.

SECTION 2

OUTLINE OF THE BASIC THEORY

2.1 COORDINATES AND BASIC NOTATION

In the development of tidal theory, spherical coordinates r, θ, φ are used with origin at the center of the planet and with r the radius vector, θ the colatitude, positive southward from the north pole, and φ the longitude, positive eastward. The corresponding wind components are w, u, and v respectively. The pressure, density, and temperature are denoted by p, ρ, T; the undisturbed values of these quantities by p_o, ρ_o, T_o; and the variations due to the tidal oscillation by δp, $\delta \rho$, δT. Some other symbols are:

a, radius of the planet

ω, angular velocity of the planet

g, acceleration of gravity on the planet

$\gamma \equiv c_p/c_v$, ratio of the specific heats of the atmosphere

$\kappa \equiv (\gamma - 1)/\gamma$

m, mean gram-molecular weight

z, height above the planetary surface

R, universal gas constant

$H \equiv RT_o/mg$, scale height of the atmosphere

χ, velocity divergence

Ω, scalar potential, describing the gravitational tide-producing force

J, heat added per unit mass and per unit time, describing the thermal tide-producing force

$2\pi/\sigma$, period of the oscillation

$f \equiv \sigma/2\omega$

Other symbols will be defined as they appear.

2.2 BASIC ASSUMPTIONS

The following assumptions are made:

(1) T_o varies with height in an arbitrary way that can be specified, but does not vary with latitude or longitude (except insofar as small periodic variations are imposed by the heating function J).

(2) p_o and ρ_o are related to T_o by the hydrostatic equation and the equation of state for an ideal gas.

(3) δp, $\delta\rho$, δT, u, v, w are small quantities whose squares and products may be neglected.

(4) Ellipticity of the planet, vertical acceleration, viscous forces, vertical variation of the radius vector, vertical variation of g, and vertical variation of $\partial\Omega/\partial z$ are all neglected.

(5) All of the dependent variables and the functions Ω and J may vary with z, θ, φ, and t. However, it is assumed that these variations are separable and may be expressed as

$$G(z, \theta, \varphi, t) = \sum_{n} G_n(z)\, \hat{G}_n(\theta) \exp[i(s\varphi + \sigma t] \qquad (2\text{-}1)$$

where G stands for any of δp, δT, $\delta\rho$, u, v, w, X, Ω or J and where s = 0, 1, 2, 3, ... Obviously, since we are dealing with linearized equations, there are many solutions of the type (2-1)(with different values of s and σ) and these solutions are additive. However, for convenience of notation, we shall often use only the subscript n and not refer explicitly to the dependence of G and $\hat{G}$ on s and σ.

2.3 FORMAL SOLUTION

The mathematical solution of the problem leads to two ordinary differential equations. One is called Laplace's tidal equation with independent variable θ and dependent variable Θ_n, where $\Theta_n = \hat{X}_n(\theta) = \hat{J}_n(\theta)$. It may be written

$$F(\Theta_n) + 4a^2\omega^2\Theta_n/gh_n = 0 \qquad (2\text{-}2)$$

where the operator F is defined by

$$F \equiv \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\sin\theta}{(f^2 - \cos^2\theta)}\frac{\partial}{\partial\theta}\right] - \frac{s}{(f^2 - \cos^2\theta)}\left[\frac{s}{\sin^2\theta} + \frac{1}{f}\left(\frac{f^2 + \cos^2\theta}{f^2 - \cos^2\theta}\right)\right] \quad (2\text{-}3$$

and where h_n is a constant with dimension of length and arises in a separation of variables.

The other differential equation is called the radial equation and in its basic form has z as independent variable and X_n as dependent variable. It also involves terms in J_n, which may be regarded as a specified function. This equation is greatly simplified by the following substitutions:

$$x \equiv \int_0^z \frac{dz'}{H(z')} \tag{2-4}$$

$$y_n(x) \exp(x/2) = X_n(z) - \kappa J_n(z)/g H(z) \tag{2-5}$$

With these, the radial equation becomes

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left[1 - \frac{4}{h_n}\left(\kappa H(x) + \frac{d H(x)}{dx}\right)\right] y_n(x) = \frac{\kappa J_n(x)}{\gamma g h_n} \exp(-x/2) \tag{2-6}$$

All of the other dependent variables can be written in terms of $\Theta_n(\theta)$ and $y_n(x)$. The expressions for u_n, v_n, w_n, and δp_n are:

$$u_n = \frac{\gamma g h_n \exp(x/2)}{4a\omega^2(f^2 - \cos^2\theta)}\left(\frac{dy_n}{dx} - \frac{y_n}{2}\right)\left(\frac{d\Theta_n}{d\theta} + \frac{s \cot\theta}{f}\Theta_n\right)\exp[i(s\varphi + \sigma t)] \tag{2-7}$$

$$v_n = \frac{i\gamma g h_n \exp(x/2)}{4a\omega^2(f^2 - \cos^2\theta)}\left(\frac{dy_n}{dx} - \frac{y_n}{2}\right)\left(\frac{\cos\theta}{f}\frac{d\Theta_n}{d\theta} + \frac{s}{\sin\theta}\Theta_n\right)\exp[i(s\varphi + \sigma t)] \tag{2-8}$$

$$w_n = \left\{ -\frac{i\sigma}{g}\,\Omega_n + \gamma h_n \exp(x/2)\left[\frac{dy_n}{dx} + \left(\frac{H(x)}{h_n} - \frac{1}{2}\right)y_n\right]\right\}\Theta_n \exp[i(s\varphi + \sigma t)] \quad (2\text{-}9)$$

$$\delta p_n = \frac{p_o(o)}{H(x)}\left[-\frac{\Omega_n}{g}\exp(-x) + \frac{\gamma h_n}{i\sigma}\exp(-x/2)\left(\frac{dy_n}{dx} - \frac{y_n}{2}\right)\right]\Theta_n \exp[i(s\varphi + \sigma t)] \quad (2\text{-}10)$$

2.4 BOUNDARY CONDITIONS AND FURTHER PROCEDURES

The study of atmospheric tides involves the solutions of (2-2) and (2-6) under various physical conditions. Solutions of Laplace's tidal equation must satisfy the conditions that u vanishes at the poles and is single-valued elsewhere. Solutions of the radial equation must satisfy the condition that $w_n = 0$ at the ground and also a second boundary condition that we shall discuss further in Section 3. The quantity h_n plays a key role in these solutions. Because it arose as a separation constant, it must have the same value in (2-2) as in (2-6), for any particular solution of the type (2-1). Its possible values are limited in one way or another by the boundary conditions and other physical conditions of the problem under consideration.

Although we are primarily concerned with forced oscillations, it is useful to consider first the problem of free oscillations ($\Omega_n = J_n = 0$). In this case, one may start with the homogeneous counterpart of (2-6). A vertical temperature distribution is assumed, which specifies H(x). For temperature distributions similar to that in the Earth's atmosphere, and for the appropriate boundary conditions, the homogeneous form of (2-6) is soluble only for one or two values of h_n. We shall refer to these as <u>atmospheric eigenvalues</u> and use the symbol $\hat{h}$ for them.

Corresponding to each atmospheric eigenvalue $\hat{h}$, there are formally a doubly infinite number of solutions of Laplace's tidal equation (2-2). These correspond first of all to different values of s (wave number). For each s there is an infinite number of solutions (modes of oscillation), each with its own period. The period for a given mode enters through the parameter f and depends of course on $\hat{h}$ and s.

Although we are not concerned here with free oscillations, the values of $\hat{h}$ corresponding to a given temperature distribution have played an important part in the discussion and interpretation of atmospheric tides for reasons that will be seen below.

In the case of tidal oscillations, the physical constraints are different. We are interested in specific periods equal to λ^{-1} of the solar (or lunar) day, ($\lambda = 1, 2, 3, \ldots.$), and therefore in particular values of f. For an oscillation characterized by particular values of λ and s, there are formally an infinite number of solutions of (2-2), each of which may be written $\Theta^s_{\lambda,n}$ ($n = 1, 2, 3, \ldots.$). Corresponding to each solution is a value of h, called an <u>equivalent depth</u>. The value of an equivalent depth $h^s_{\lambda,n}$ depends on λ, s, and n but not on the vertical temperature distribution. A solution of (2-6) is then sought for the equivalent depth corresponding to the mode of oscillation under consideration. Contrary to the case of free oscillations, solutions of (2-6) are <u>not</u> restricted to values of h corresponding to the atmospheric eigenvalues, because either Ω_n or J_n is different from zero. However, if an oscillation $\Theta^s_{\lambda,n}$ has associated with it an equivalent depth $h^s_{\lambda,n}$ that is nearly equal to one of the atmospheric eigenvalues $\hat{h}$, and if further the corresponding forcing function $\Omega^s_{\lambda,n}$ (or $J^s_{\lambda,n}$) is not too small, then one may expect a large amplitude for that mode of oscillation. Such a phenomenon is referred to as <u>resonance</u>. There are various ways of quantifying this rather vague definition of resonance, and we shall return to this in Section 4.

SECTION 3

METHODS AND RESULTS OF SOLVING THE DIFFERENTIAL EQUATIONS

In the last part of the previous section we discussed qualitatively the procedures for investigating free or forced oscillations. Here we shall go into more detail with respect to tidal oscillations, adapting solutions that have been obtained for our atmosphere to that of Mars.

3.1 LAPLACE'S TIDAL EQUATION, EARTH'S ATMOSPHERE

Laplace's tidal equation (2-2) does not involve the thermal structure or composition of the atmosphere and its solutions are easily applicable to the Martian atmosphere. These solutions are called Hough's functions and have the form of a series of associated Legendre functions of argument $\mu = \cos\theta$. Specifically,

$$\Theta^{s}_{\lambda,n}(\mu) = \sum_{\nu} C^{s,\nu}_{\lambda,n} \; P^{s}_{\nu}(\mu) \tag{3-1}$$

where

	$n = 1, 2, 3, \ldots.$	for $s = 0$
	$n = s, s+1, s+2, \ldots.$	for $s \neq 0$
	$\nu = 1, 3, 5, \ldots.$	for $s = 0$, n odd
	$\nu = 2, 4, 6, \ldots.$	for $s = 0$, n even

$\nu = s, s+2, s+4, \ldots.$ for $s \neq 0$, $(n-s)$ even

$\nu = s+1, s+3, s+5, \ldots.$ for $s \neq 0$, $(n-s)$ odd

The determination of $h^s_{\lambda,n}$ and computation of the coefficients $C^{s,\nu}_{\lambda,n}$ for each λ, n, s are rather time-consuming. Siebert has given results for modes of oscillation that are of interest in the Earth's atmosphere. Although these will not be reproduced here, we show as an example some of the results for an oscillation of period one half a solar day when s = 2, which is the primary oscillation in the Earth's atmosphere:

$\underline{s = 2,\ \lambda = 2,\ f = .99727,\ \sigma = 1.4544 \times 10^{-4}\ \text{sec}^{-1}}$

$n = 2$, $h^2_{2,2} = 7.85$ km

$$\Theta^2_{2,2} = P^2_2 - 0.339\ P^2_4 + 0.041\ P^2_6 - 2 \times 10^{-3}\ P^2_8 + \ldots.$$

$n = 4$, $h^2_{2,4} = 2.11$ km

$$\Theta^2_{2,4} = 0.202\ P^2_2 + P^2_4 - 0.819\ P^2_6 + 0.24\ P^2_8 - 0.04\ P^2_{10} + \ldots.$$

From the properties of the Legendre functions and the specifications of ν listed under (3-1), it is apparent that Hough's functions are symmetric about the equator when $(n - s)$ is even and anti-symmetric when $(n - s)$ is odd. Only the first two symmetric modes are given above for $s = 2$, $\lambda = 2$. Usually, but not always, the leading term in a Hough's function is the one for which $\nu = n$. For given values of s and λ, $h^s_{\lambda,n}$ decreases as n increases.

These general properties of the solutions will suffice for our further discussion.

3.2 LAPLACE'S TIDAL EQUATION, MARTIAN ATMOSPHERE

For the Martian atmosphere, we are interested in oscillations of period one (Martian) solar day or a sub-multiple thereof. This period enters into Laplace's tidal equation only through the parameter $f = \sigma/2\omega$. By definition, $\sigma = 2\pi\lambda/D$ and $\omega = 2\pi/D'$ where D is the length of a solar day and D' is the length of a sidereal day. Accordingly, $f = (\lambda/2)(D'/D)$. For our atmosphere $D'/D = .99725$, while for Mars $D'/D = .9985$. For our purposes this is a negligible difference and we may assume that the Hough's functions for a λ^{-1} oscillation are the same in both atmospheres.

The equivalent depth corresponding to a given mode of oscillation enters only in the factor $4a^2\omega^2/gh_n$. We may assume that this factor is essentially the same in the two atmospheres for a given mode of oscillation. Nevertheless, values of the equivalent depth h_n differ appreciably because the planetary constant $a^2\omega^2/g$ differs between Earth and Mars. Specifically

$$h^s_{\lambda,n}\ (\text{Mars}) = .74\ h^s_{\lambda,n}\ (\text{Earth})\ .$$

Table 3-1 gives values of $h^s_{\lambda,n}$ for the two planets for some modes of oscillation that are of interest.

TABLE 3-1

Values of $h^{s}_{\lambda,n}$ for Earth and Mars.

	$h^{s}_{\lambda,n}$ (Earth) (km)	$h^{s}_{\lambda,n}$ (Mars) (km)
$s = 0, \lambda = 2, n = 2$	8.85	6.5
$s = 0, \lambda = 2, n = 4$	2.21	1.6
$s = 1, \lambda = 1, n = 1$	0.63	0.47
$s = 2, \lambda = 2, n = 2$	7.85	5.8
$s = 2, \lambda = 2, n = 4$	2.11	1.6
$s = 3, \lambda = 3, n = 3$	12.89	9.5
$s = 3, \lambda = 3, n = 4$	7.66	5.7

3.3 THE RADIAL EQUATION, EARTH'S ATMOSPHERE

Let us consider first the homogeneous counterpart of (2-6), since its solution reveals the atmospheric eigenvalues and furthermore is required in the general solution of (2-6). This is

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left[1 - \frac{4}{h_n}\left(\kappa H(x) + \frac{dH(x)}{dx}\right)\right] y_n(x) = 0 \qquad (3\text{-}2)$$

This equation is to be solved subject to the boundary condition that $w_n = 0$ at $x = 0$, w_n being given by (2-9). A second boundary condition has been attained in a variety of ways by different authors. A natural condition to impose is that the kinetic energy of tidal motion per column of unit cross section shall remain finite. This reduces to the mathematical expression

$$\lim_{x \to \infty}\left[y_n(x) \cdot x^{\frac{1}{2}}\right] = 0 \qquad (3\text{-}3)$$

However, for certain models of the vertical temperature distribution, this condition gives the result that there can be no oscillations for values of h_n below some critical value. This result is not verified by observation and is undoubtedly due to the failure of certain assumptions (for example, neglect of heat conduction and non-linear terms) at very high levels. Therefore it is customary to admit solutions that do not satisfy (3-3), but instead give an upward flow of energy (Wilkes, 1949) or a zero vertical flow of energy (Siebert, 1961) at high levels.

In treating Equation (3-2), one must first specify a vertical temperature distribution to define H(x). In the general case, (3-2) is then solved numerically. Functional solutions are possible for an isothermal atmosphere or for an atmosphere with linear lapse rate or for a special exponential atmosphere to be described later.

The history of studies of (3-2) is of considerable interest and has important implications for a study of the Martian atmosphere. Pekeris (1937) first pointed out that for a vertical temperature distribution that appeared reasonable at the time our atmosphere might have two eigenvalues, one near 10 km and another near 8 km. The first had been inferred by Taylor (1929) from the observed velocity of propagation of the pressure wave generated by the Krakatoa eruption. The second agreed closely enough with the equivalent depth for the $\Theta^2_{2,2}$ oscillation to suggest a very important resonance effect. Weekes and Wilkes (1947) (see also Wilkes, 1949) pursued this idea with more extensive computations. They also pointed out that the eigenvalue of about 10 km is associated with the temperature distribution in the upper troposphere and lower stratosphere and is little affected by conditions at higher levels. The second eigenvalue of about 8 km is associated with the temperature distribution in the upper stratosphere and mesosphere. Figure 1 shows, after Wilkes (1949), some vertical distributions of temperature that would explain the observed $\Theta^2_{2,2}$ oscillation as an oscillation forced to an important degree by the solar gravitational tidal force and magnified by resonance.

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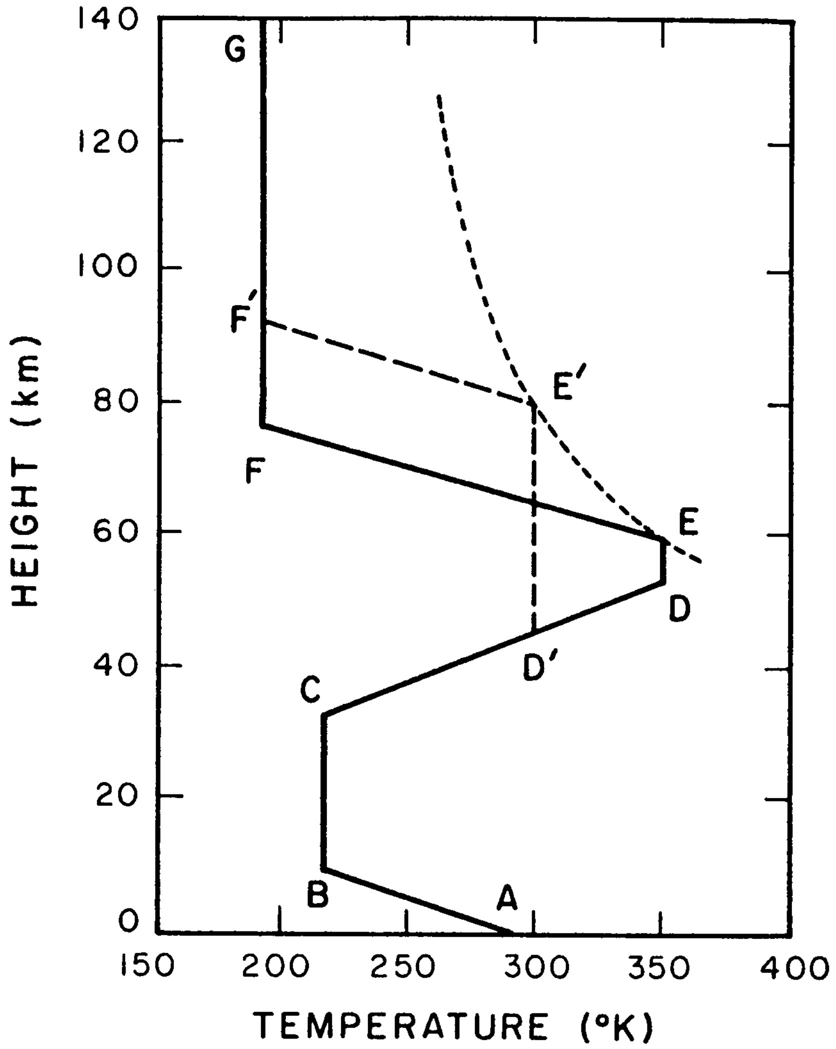

Figure 1. Vertical distributions of temperature in our atmosphere that would provide sufficient resonance to explain the solar semi-diurnal tide as a gravitational oscillation. Any profile of the form A B C D' E' F' G (where D' lies between C and D, E' lies vertically above it on the indicated curve, and F' lies between F and G) would suffice. (After Wilkes, 1949).

The trouble with this attractive explanation, which was widely accepted in 1950, is that temperature data obtained with the use of rockets in recent years fail to verify the required temperature distribution. Computations by Jacchia and Kopal (1952) and by Sen and White (1955) show that our atmosphere with its observed temperature distribution does not amplify the solar gravitational tide to anywhere near the degree required by observation.

In this situation, the explanation of the relatively large $\Theta_{2,2}^{2}$ oscillation has been sought in terms of a thermal tide, driven by periodic heating of the atmosphere and requiring only a moderate amount of amplification. Mathematically, this requires a description of the heating function in terms of J_n that appears in Equation (2-6). The general solution of (2-6) then includes a particular solution as well as the solutions of the homogeneous Equation (3-2). In the general case, the entire solution may be obtained numerically.

Various heating models have included surface heating communicated to the atmosphere by eddy conduction (Sen and White, 1955), absorption of solar radiation by water vapor (Siebert, 1961) and absorption of solar radiation by ozone (Butler and Small, 1963). Although serious problems are involved in specifying the heating functions, these mechanisms appear to be capable of explaining the magnitudes of the observed tides. It seems quite likely that a satisfactory explanation is presently evolving along these lines.

3.4 THE RADIAL EQUATION, MARTIAN ATMOSPHERE

In applying tidal theory to the Martian atmosphere, one is faced first of all with the problem of estimating a vertical temperature distribution. Naturally, there is a great deal of uncertainty about this factor. However, various estimates have been made on the basis of radiative considerations. Unless the composition of the Martian atmosphere is considerably different than is presently inferred, especially in amounts of carbon dioxide, water vapor, and ozone (or by containing an unsuspected constituent that is radiatively active), these estimates must establish the broad outlines of the distribution. A recent temperature profile prepared by Rasool (1963) is shown in Figure 2, being a composite of estimates by Arking (1962) and Chamberlain (1962).

On the basis of our experience with the terrestrial atmosphere, we should expect to find an eigenvalue associated with the temperature decrease in the troposphere and the low temperature in the stratosphere. Another might be associated with the rather steep lapse rate just above 100 km and the temperature minimum at the mesopause. However, arguing again on the basis of our terrestrial experience, we could expect the latter to depend rather critically on the details of the temperature distribution near the mesopause and these are certainly not known.

For a first appraisal of the Martian tidal problem, it would seem reasonable to consider only the effects of the troposphere and stratosphere, which in our atmosphere give rise to the eigenvalue of about 10 km. For

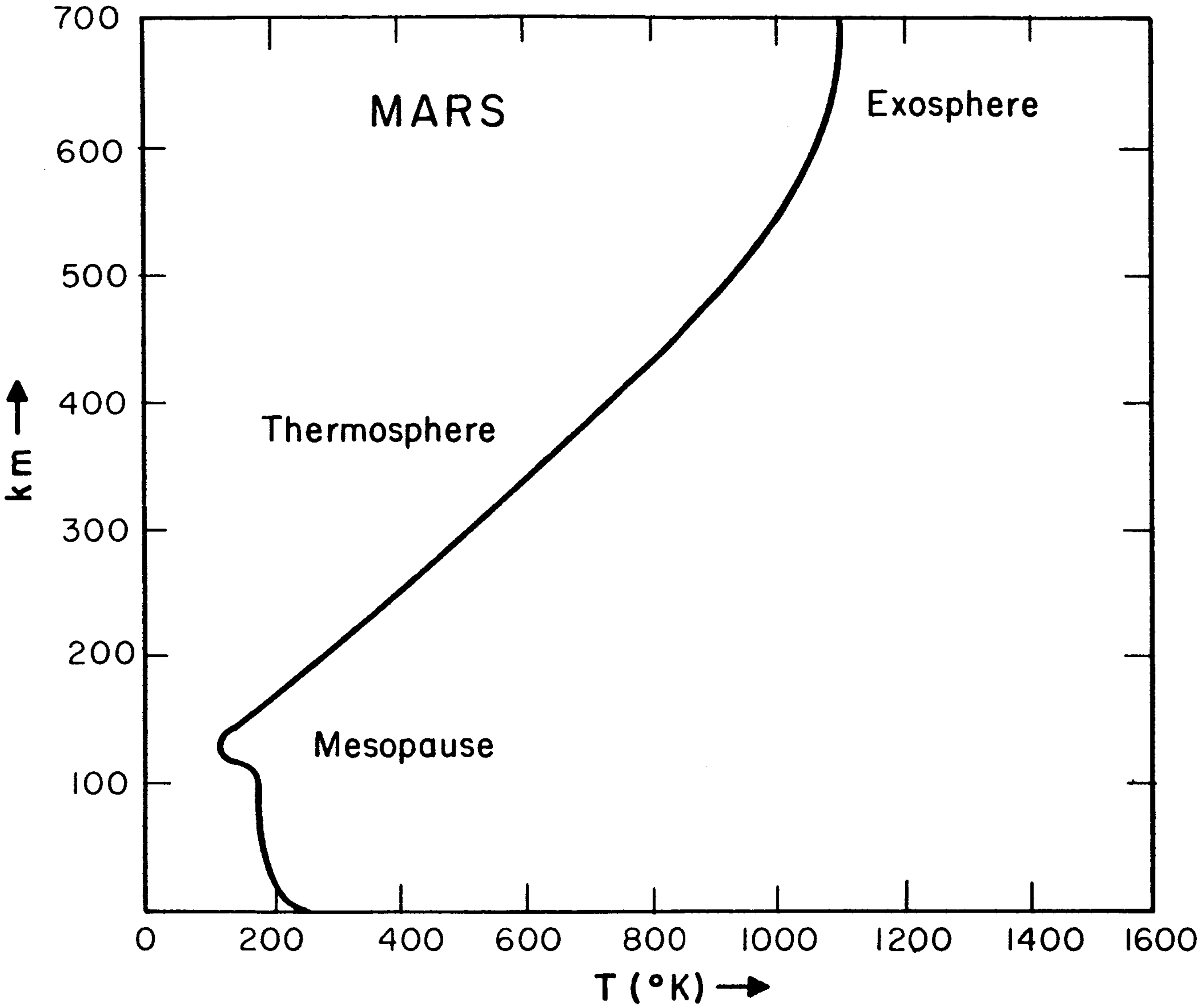

Figure 2. Possible vertical distribution of temperature in the Martian atmosphere according to Rasool (1963).

this first appraisal, and especially in view of the large degree of uncertainty about the actual Martian temperature distribution, it is also reasonable to use a rather simple model that is capable of yielding analytic solutions. In this way the effects of varying environmental parameters can be easily studied.

Such a simple model is available (Siebert, 1961) in the form of a special exponential temperature distribution. Specifically, let

$$T_o(x) = \left[T_o(o) - T_o(\infty)\right] \exp(-\kappa x) + T_o(\infty) \tag{3-4}$$

in which $T_o(o)$ and $T_o(\infty)$ are adjustable constants which correspond in the model to surface temperature and temperature of the isothermal top. This distribution makes it possible to derive relatively simple expressions for the study of all relevant tidal problems.

Figure 3 shows a few of the estimates that have been made of the vertical temperature distribution in the lowest 40 km of the Martian atmosphere. Figure 3 also shows three vertical distributions of temperature according to (3-4) for $T_o(o) = 230°K$ and $T_o(\infty) = 160°K$ and $80°K$. For further orientation, Figure 4 shows the temperature distribution in the terrestrial troposphere and stratosphere and a representation of it by (3-4). In the case of our atmosphere this representation is adequate to describe the resonance characteristics associated with the eigenvalue of about 10 km. We should expect that the same will be true in the Martian atmosphere, as long as this eigenvalue does not need to be very accurately known.

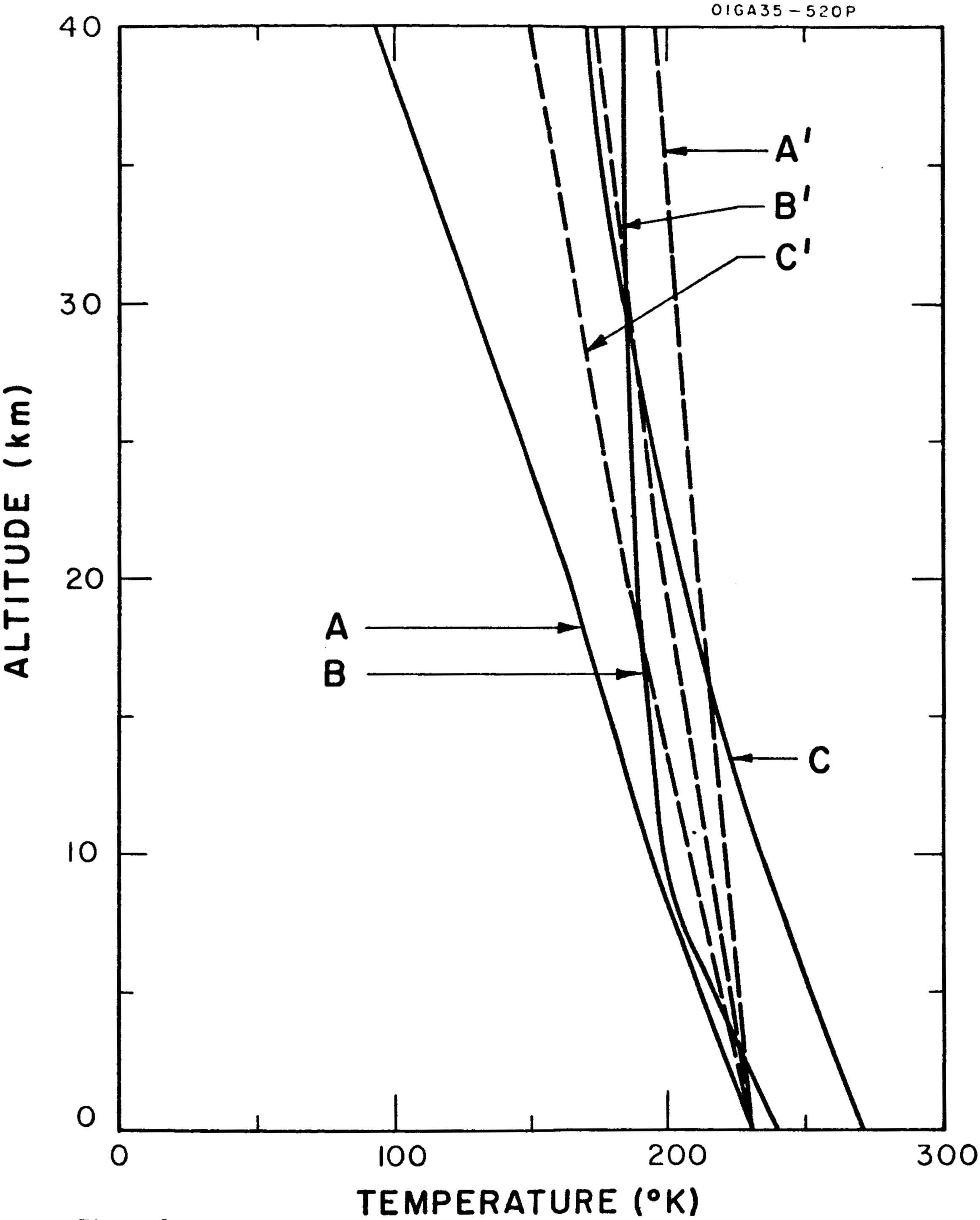

Figure 3. Possible vertical distributions of temperature in the lowest 40 km of the Martian atmosphere. Full curves show estimates by: A, Ohring (1963); B, Rasool (1963); C, Goody (1957). Dashed curves show representation by model atmosphere with $T_o(o) = 230°K$ and: A', $T_o(\infty) = 160°K$; B', $T_o(\infty) = 120°K$; C', $T_o^o = 80°K$.

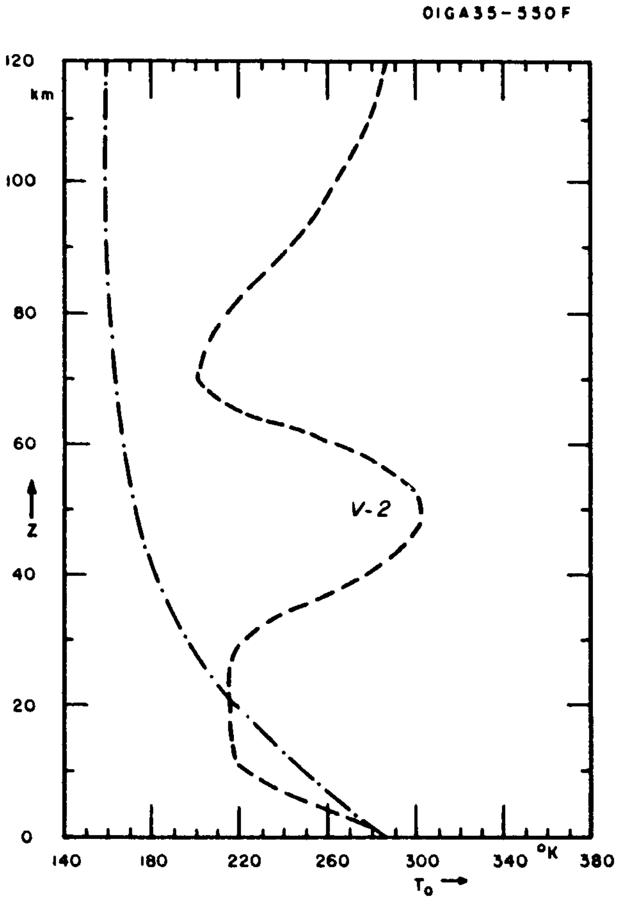

Figure 4. Vertical distribution of temperature in the Earth's atmosphere according to rocket measurements (curve marked V-2) and as represented by model atmosphere leading to essentially the same atmospheric eigenvalue. (After Siebert, 1961).

The eigenvalue of the Martian atmosphere for a temperature distribution like (3-4) is easily determined. Substituting (3-4) in (3-2) gives

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left[1 - \frac{4\kappa H(\infty)}{h_n}\right] y_n = 0 \ . \tag{3-5}$$

Assuming for the moment that $h_n > 4\kappa H(\infty)$ we note that the solution that satisfies the upper boundary condition (3-3) is

$$y_n(x) = A_n \exp\left\{ - \frac{x}{2}\sqrt{1 - \frac{4\kappa H(\infty)}{h_n}} \right\} \ . \tag{3-6}$$

At the lower boundary, set $w_n = 0$, using for w_n the expression (2-9) with $\Omega_n = 0$. This implies a free oscillation for which h_n is an eigenvalue $\hat{h}$. The result is

$$\hat{h} = \frac{H^2(o)}{H(o) - \kappa H(\infty)} = \frac{R}{mg} \frac{T_o^2(o)}{T_o(o) - \kappa T_o(\infty)} \ . \tag{3-7}$$

Note from this solution that $\hat{h} > 4\kappa H(\infty)$ if $H(o) > \kappa H(\infty)$, which is true for all the atmospheres we shall consider.

If, as is generally assumed, the Martian atmosphere is predominantly nitrogen, then its mean molecular weight is slightly less and its specific heats (per unit mass) slightly greater than those of our atmosphere. However, we shall assume throughout that R/m, c_p and c_v have the same values in the two atmospheres. The acceleration of gravity is known to be considerably less on Mars and we use the value 3.8×10^2 cm sec^{-2}. Table 3-2 gives some eigenvalues for the Martian atmosphere when its temperature distribution is represented by (3-4) and various values of $T_o(o)$ and $T_o(\infty)$ are assumed.

One sees immediately that, owing to the small value of g on Mars, these are larger than their terrestrial counterpart, which is about 10 km. We noted earlier (Table 3-1) that the equivalent depths of the periodic oscillations are smaller than their terrestrial counterparts. We can draw two conclusions from these preliminary results before going on to more detailed considerations:

(1) Resonance effects due to this eigenvalue are smaller in the Martian atmosphere than in Earth's.

(2) The exact vertical temperature distribution assumed for the Martian troposphere and lower stratosphere is not a critical factor in order-of-magnitude tidal studies. This is because the oscillations of interest have periods so far removed from the probable free period of the atmosphere that their behavior is not very sensitive to the exact value of that free period.

It is important to note again for emphasis that the above conclusions refer to a simplified temperature distribution capable of exhibiting only one eigenvalue. Should the temperature distribution at high levels over Mars be such that a second eigenvalue appears, they would not necessarily apply to this second eigenvalue. A better knowledge of the Martian temperature distribution as well as more elaborate computations would be required to test this possibility.

TABLE 3-2

Values of the atmospheric eigenvalue (in km) for the model Martian atmosphere and various values of $T_o(o)$ and $T_o(\infty)$

$T_o(o)$ \ $T_o(\infty)$	160°K	120°K	80°K
250°K	23.2	22.0	20.9
230°K	21.8	20.5	19.4
210°K	20.4	19.0	17.9

SECTION 4

THE AMPLITUDES OF TIDAL OSCILLATIONS IN THE MARTIAN ATMOSPHERE

4.1 THE SOLAR GRAVITATIONAL TIDE

The solar gravitational tide in the Martian atmosphere must be completely insignificant unless there is a high degree of resonance for one of the oscillations. We have already noted above that such resonance is not to be expected from the principal atmospheric eigenvalue. The purpose of the present discussion is to make these assertions more quantitative.

With $J_n = 0$, (3-6) is a solution of (2-6) that satisfies the upper boundary condition (3-3) when $h_n > 4\kappa H(\infty)$. For $h_n < 4\kappa H(\infty)$, (3-3) cannot be satisfied. Following Siebert, we take as a solution in this case

$$y_n(x) = A_n(\cos \tfrac{1}{2} \tilde{\beta}_n x + \sin \tfrac{1}{2} \tilde{\beta}_n x) \tag{4-1}$$

where

$$\tilde{\beta}_n = \sqrt{\frac{4\kappa H(\infty)}{h_n} - 1} \tag{4-2}$$

This solution gives zero vertical flow of energy at high levels and provides continuity at the point in the resonance curve (to be defined below) where the two solutions (3-6) and (4-1) join.

Application of the lower boundary condition allows the A_n in either (3-6) or (4-1) to be determined. The result may be written

$$A_n = \frac{2i\sigma\Omega_n(o)}{\gamma g[2H(o) - h_n(1 + b_n)]} \tag{4-3}$$

where

$$b_n = \sqrt{1 - \frac{4\kappa H(\infty)}{h_n}} \equiv \beta_n \qquad \text{for } h_n > 4\kappa H(\infty)$$

$$b_n = -\sqrt{\frac{4\kappa H(\infty)}{h_n} - 1} \equiv -\tilde{\beta}_n \qquad \text{for } h_n < 4\kappa H(\infty)$$

A convenient way to describe a tidal oscillation is in terms of the pressure oscillation at the ground. I can be shown from (2-9) and (2-10) that

$$\delta p_n(o) = \frac{i\gamma}{\sigma} p_o(o) y_n(o) \ . \tag{4-4}$$

Therefore in the present case,

$$\delta p_n(o) = -\frac{2\ \Omega_n(o)\rho_o(o)H(o)}{2\ H(o) - h_n(1 + b_n)} \ . \tag{4-5}$$

The "equilibrium tide" $\overline{\delta p_n(o)}$ is given by the first term of (2-10) as $-\rho_o(o)\,\Omega_n(o)$. This is used as a unit to express the pressure oscillation that finally appears in the atmosphere. If M_n represents the resonance effect,

$$M_n \equiv \frac{\delta p_n(o)}{\overline{\delta p_n(o)}} = \frac{2\,H(o)}{2\,H(o) - h_n(1 + b_n)} . \tag{4-6}$$

A plot of M_n as a function of h_n is called a resonance curve. In place of such a curve, which would show strong magnification only near 20 km for the present simple model, Table 4-1 gives the values of M_n for oscillations whose equivalent depths are listed in Table 3-1. For this purpose, we take $T_o(o) = 230°K$ and $T_o(\infty) = 120°K$; but other choices within what appear to be reasonable limits would make no appreciable difference in the results. The magnification factor is near 1 for all oscillations. In the earth's atmosphere corresponding values for the lunar tides are of the same magnitude (although somewhat larger); and that tide, involving much greater gravitational force than the solar tide on Mars, is only barely detectable. It is therefore clear, as anticipated, that a noticeable Martian tide, if there is any, must be thermally driven.

4.2 THE THERMAL TIDE, SURFACE HEATING

To study the thermal tide due to surface heating of Mars, we first express the heating function J in terms of the temperature variation by

$$J = c_p \frac{\partial \tau}{\partial t} \tag{4-7}$$

TABLE 4-1

Amplitude of the gravitational surface pressure oscillation on Mars for model atmosphere with $T_o(o) = 230°K$ and $T_o(\infty) = 120°K$

	$h^s_{\lambda,n}$ (km)	$M^s_{\lambda,n}$
$s = 0,\ \lambda = 2,\ n = 2$	6.5	1.0
$s = 0,\ \lambda = 2,\ n = 4$	1.6	0.94
$s = 1,\ \lambda = 1,\ n = 1$	0.47	0.96
$s = 2,\ \lambda = 2,\ n = 2$	5.8	1.0
$s = 2,\ \lambda = 2,\ n = 4$	1.6	0.94
$s = 3,\ \lambda = 3,\ n = 3$	9.5	1.2
$s = 3,\ \lambda = 3,\ n = 4$	5.7	1.0

where τ is that part of the observed temperature variation due to the heating. That is, it does not include adiabatic temperature changes that might be associated with a resulting tidal oscillation. For our purposes, the distinction is unimportant.

If we write τ as

$$\tau = \sum_n \tau_n(z)\, \Theta_n(\theta)\, \exp[i(s\varphi + \sigma t)] \tag{4-8}$$

then

$$J_n(z) = c_p i\sigma\tau_n(z) = \frac{i\sigma R}{m\,\kappa}\, \tau_n(z) \, . \tag{4-9}$$

Let us assume, as is customary in studies of the Earth's atmosphere, that the diurnal variation of temperature at the surface is known and that the temperature variation at higher levels results from a vertical transfer of heat by eddy conduction. Thus

$$J = c_p K \frac{\partial^2 \tau}{\partial z^2} \tag{4-10}$$

which with (4-9) gives the differential equation

$$\frac{d^2 \tau_n}{dz^2} - \frac{i\sigma}{K}\, \tau_n = 0 \, . \tag{4-11}$$

Here K is a coefficient of eddy conduction which will be assumed constant. This is a rather crude assumption, but should suffice to give answers with the correct order of magnitude. The solution of (4-11), with appropriate boundary conditions, is

$$\tau_n(z) = \tau_n(o) \exp[-kz/H(o)] \tag{4-12}$$

where

$$k = H(o) \sqrt{\frac{\sigma}{K}} \exp(\pi i/4)$$

In Equation (2-6) it is necessary to express J_n as a function of x and not z. In the present model atmosphere,

$$z = \frac{H(o) - H(\infty)}{\kappa} [1 - \exp(-\kappa x)] + H(\infty)x .$$

Since $\kappa < 1$ and for the part of the atmosphere that is heated by the ground $x \ll 1$, we can write $\exp(-\kappa x) \cong 1 - \kappa x$ and it follows that

$$\tau_n(x) = \tau_n(o) \exp(-kx) .$$

Therefore according to (4-9), the appropriate heating function is

$$J_n(x) = \frac{i\sigma R}{m\kappa} \tau_n(o) \exp(-kx) . \tag{4-13}$$

Inserting (4-13) in (2-6) gives finally

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left[1 - 4\frac{\kappa\, H(\infty)}{h_n}\right] y_n = \frac{i\sigma H(o)}{\gamma h_n T_o(o)}\, \tau_n(o)\, \exp\left[-\frac{(1+2k)x}{2}\right]. \quad (4\text{-}14)$$

The homogeneous part of the solution of (4-14) has already been given by (3-6) or (4-1). A particular solution is

$$\tilde{y}_n = B_n \exp\left[-\frac{(1+2k)x}{2}\right] \quad (4\text{-}15)$$

where

$$B_n = \frac{i\sigma\, H(o)\, \tau_n(o)}{\gamma\, T_o(o)\, [(1+k)kh_n + \kappa\, H(\infty)]}\,. \quad (4\text{-}16)$$

It is next necessary to determine the constant A_n by application of the boundary condition at the ground. When this is done and the general solution is used to determine $\delta p_n(o)$, one obtains

$$\delta p_n(o) = -\frac{p_o(o)\, H(o)\, h_n(1 + 2k - b_n)\, \tau_n(o)}{T_o(o)[2\, H(o) - h_n(1+b_n)][(1+k)kh_n + \kappa\, H(\infty)]}\,. \quad (4\text{-}17)$$

For computational purposes, this can be simplified considerably. For all situations that will be investigated $k \gg 1$, $2k \gg b_n$, and $k^2 h_n \gg \kappa\, H(\infty)$, so that

$$\delta p_n(o) \cong \frac{2\ p_o(o)\ \tau_n(o)}{T_o(o)\,[2\ H(o) - h_n(1 + b_n)]}\sqrt{\frac{K}{\sigma}}\ \exp(3\pi i/4)\ . \tag{4-18}$$

For all oscillations listed in Table 4-1, and for any values of H(o) and H(∞) that appear reasonable, the factor in square brackets in the denominator of (4-18) has a value in the neighborhood of 30-40 km. There is therefore no preferential excitation of a particular period and one would expect the diurnal oscillation to predominate simply because the amplitude of the diurnal temperature oscillation is in all likelihood larger than that of the semi-diurnal, ter-diurnal, etc. For this oscillation, $\sigma \cong 7 \times 10^{-5} sec^{-1}$. Taking $K = 10^5 cm^2 sec^{-1}$, which would be a rather large average value of K for our atmosphere, one gets from (4-18) the approximate relation

$$\frac{\delta p_n(o)}{p_o(o)} \cong \frac{1}{75}\ \frac{\tau_n(o)}{T_o(o)}\ . \tag{4-19}$$

For example, if the amplitude of the diurnal temperature variation near the equator and at the ground is 30°C (Ohring, Tang, and DeSanto, 1962), then $\delta p_n(o)/p_o(o) \cong 1/600$ which gives $\delta p_n(o) \cong 0.1$ to 0.2 mb for the generally accepted surface pressure of something near 100 mb.

Relative to surface pressure, this amplitude of the diurnal pressure oscillation on Mars is of the same order of magnitude as the amplitude of the observed semi-diurnal oscillation on Earth. It is small enough to indicate that a thermal tide on Mars driven by surface heating is not an important factor in the circulation of that planet. It is unlikely that the estimates of $\tau_n(o)$, K, and the denominator of (4-18) which lead to this conclusion are sufficiently inaccurate to invalidate this general result. It might, however, be incorrect if the temperature distribution on Mars were such as to give a second atmospheric eigenvalue rather close to one of the equivalent depths listed in Table 3-1.

4.3 THE THERMAL TIDE, ATMOSPHERIC ABSORPTION

There is every indication that the observed solar tide in our atmosphere is driven principally by heating of the atmosphere caused by absorption of solar radiation by water vapor (Siebert, 1961) or by ozone (Butler and Small, 1963). Although the amplitude of these thermal effects is small relative to temperature variations near the surface, the latter extend only through the lowest several hundred meters of the atmosphere and thus have less effect.

Until the composition of the Martian atmosphere is known with more reliability, it is not possible to draw conclusions about direct solar heating. Water vapor and ozone are known to be less abundant than in our atmosphere (Ohring, 1963), but only reasonable upper limits can be specified. The present study of this effect will be confined to the discussion of a solution given by Siebert for a temperature variation whose vertical distribution is unspecified.

With the help of (4-9), the radial equation (2-6) becomes, for the model atmosphere being considered,

$$\frac{d^2 y_n}{dx^2} - \frac{1}{4}\left(1 - 4\kappa \frac{H(\infty)}{h_n}\right) y_n(x) = C_n \tau_n(x) \exp(-x/2) \qquad (4\text{-}20)$$

where

$$C_n \equiv \frac{i\sigma}{\gamma h_n} \frac{H(o)}{T_o(o)}$$

As before the homogeneous part of the solution of (4-20) is given by (3-6) or (4-1). Complete solutions of (4-20) corresponding to (3-6) and (4-1) respectively and satisfying the upper boundary condition are:

$$y_n(x) = A_n \exp(-\beta_n x/2) + \frac{C_n}{\beta_n}\left[\exp(\beta_n x/2)\int_\infty^x \exp(-\beta_n x/2)\exp(-x/2)\tau_n dx - \exp(-\beta_n x/2)\int_\infty^x \exp(\beta_n x/2)\exp(-x/2)\tau_n dx\right] \qquad (4\text{-}21)$$

$$y_n(x) = A_n\left(\cos\frac{\tilde{\beta}_n x}{2} + \sin\frac{\tilde{\beta}_n x}{2}\right) + \frac{2C_n}{\tilde{\beta}_n}\left[\sin\frac{\tilde{\beta}_n x}{2}\int_\infty^x \cos\frac{\tilde{\beta}_n x}{2}\exp(-x/2)\tau_n dx - \cos\frac{\tilde{\beta}_n x}{2}\int_\infty^x \sin\frac{\tilde{\beta}_n x}{2}\exp(-x/2)\tau_n dx\right] \qquad (4\text{-}22)$$

The arbitrary constant A_n can be determined from the lower boundary condition, and the expression for the amplitude of the surface pressure oscillation is

$$\delta p_n(o) = \frac{-\,2\,p_o(o)\,H(o)\displaystyle\int_o^\infty f_n(x)\,\exp(-x/2)\,\tau_n(x)\,dx}{T_o(o)\,[2\,H(o) - h_n(1 + b_n)]} \tag{4-23}$$

where

$$f_n(x) = \exp(-\beta_n x/2) \qquad , \text{ for } h_n > 4\kappa\, H(\infty)$$

$$f_n(x) = \cos\frac{\tilde{\beta}_n x}{2} + \sin\frac{\tilde{\beta}_n x}{2} \qquad \text{for } h_n < 4\kappa\, H(\infty) \ .$$

For the oscillations that are likely to be important and for reasonable values of $H(o)$ and $H(\infty)$, the factor $2H(o)/[2\,H(o) - h_n(1 + b_n)]$ is of order of magnitude 1. Therefore the equation analogous to (4-19) is

$$\frac{\delta p_n(o)}{p_o(o)} \cong -\frac{1}{T_o(o)}\int_o^\infty f_n(x)\,\exp(-x/2)\,\tau_n(x)\,dx \ . \tag{4-24}$$

The integral represents the contributions of temperature variations caused by diabatic processes at all levels, weighted in a certain way that depends on the particular model atmosphere and oscillation under consideration. Although a numerical application of (4-24) to the Martian atmosphere is probably unjustified at present by our lack of knowledge of the temperature variation, nevertheless the vertical variation of the weighting function $f_n(x)\ \exp(-x/2)$ for various oscillations is of some interest. Figure 5 shows this for the $\Theta^1_{1,1}$ and $\Theta^2_{2,2}$ oscillations (for the model atmosphere with $T_o(o) = 230°K$ and $T_o(\infty) = 120°K$)

Of special interest is the behavior of $f_n(x)\ \exp(-x/2)$ for $\lambda = 1$, $s = 1$, $n = 1$. This function changes sign at about 17 km and again at about 37 km. This behavior indicates qualitatively that if heating took place throughout a deep layer of the Martian atmosphere, as might conceivably happen if ozone is present (Ohring and Coté, 1963), then its effect in exciting the $\Theta^1_{1,1}$ oscillation would be inhibited by the resulting cancellation effect. On the other hand, the weighting factor for the $\Theta^2_{2,2}$ oscillation retains the same sign to over 50 km; and if there were an appreciable value of $\tau^2_{2,2}$ through a deep layer of the atmosphere the semi-diurnal oscillation might be preferentially excited. Although the details are different in our atmosphere, this is the sort of explanation that is currently emerging for the relatively large $\Theta^2_{2,2}$ oscillation in our atmosphere (Butler and Small, 1963).

An order-of-magnitude assessment of the effect of atmospheric heating may be obtained by assuming that $\tau_n(x)$ has some constant value in the region between the ground and an upper level x. Then for the solution given by (4-22), which must be used for all the oscillations listed in

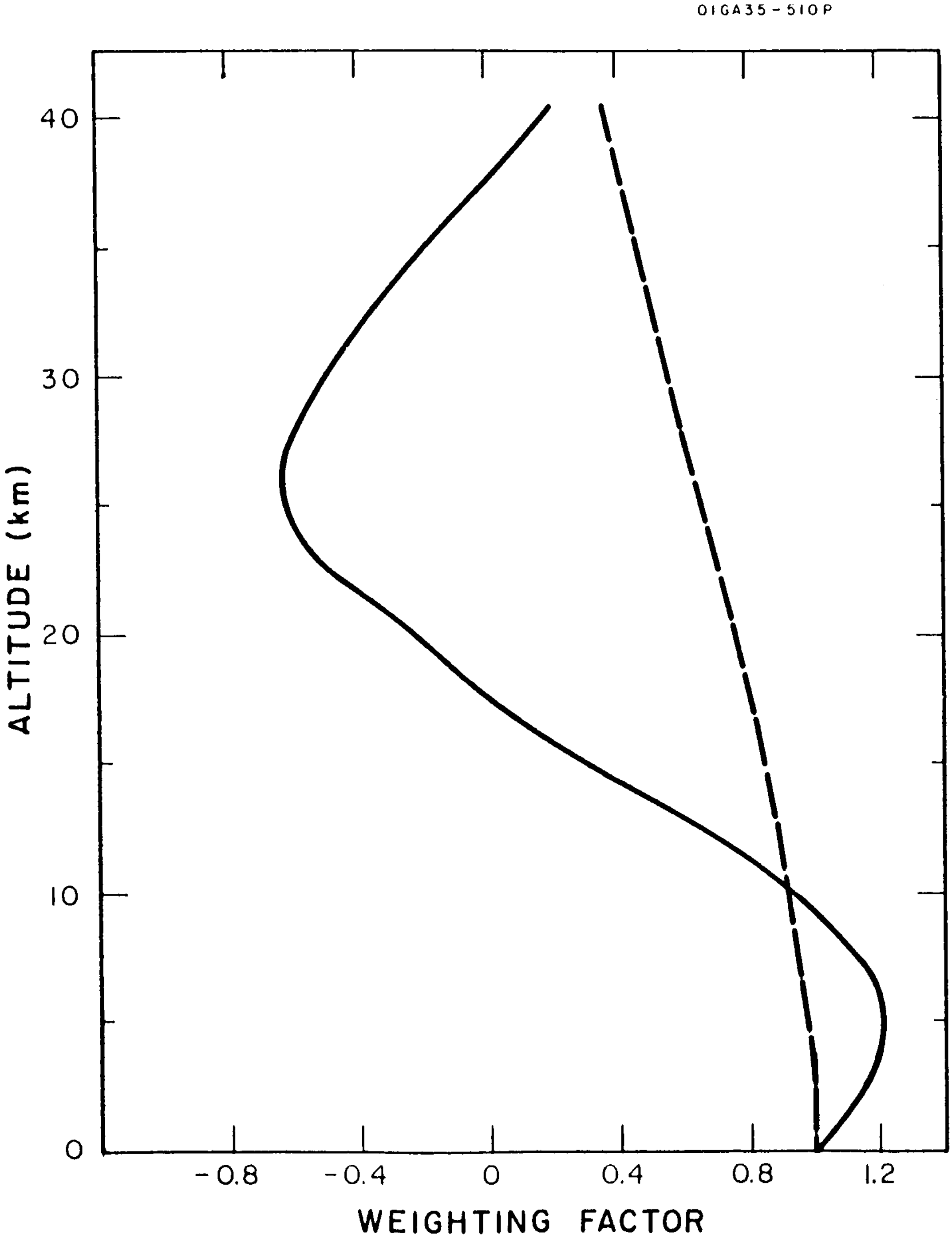

Figure 5. Variation with altitude of the weighting factor for the heating function. Full curve is the weighting function for a diurnal oscillation, $\lambda = 1$, $s = 1$, $n = 1$. Dashed curve is the weighting function for a semi-diurnal oscillation, $\lambda = 2$, $s = 2$, $n = 2$.

Table 3-1 when $T_o(\infty)$ has reasonable values,

$$\tau_n \int_0^x f_n(x)\exp(-x/2)\,dx = \frac{2}{\tilde{\beta}_n^2 + 1}\left[\tilde{\beta}_n + 1\right] + \frac{2\exp(-x/2)}{\tilde{\beta}_n^2 + 1}\left[\left(\tilde{\beta}_n - 1\right)\sin\frac{\tilde{\beta}_n x}{2}\right. \tag{4-25}$$

$$\left. - \left(\tilde{\beta}_n + 1\right)\cos\frac{\tilde{\beta}_n x}{2}\right] .$$

For $T_o(\infty) = 120°K$, $\tilde{\beta}_n = 4.63$ for $\lambda = 1$, $s = 1$, $n = 1$; $\tilde{\beta}_n = .90$ for $\lambda = 2$, $s = 2$, $n = 2$; and $\tilde{\beta}_n = .33$ for $\lambda = 3$, $s = 3$, $n = 3$. If the second term in (4-25) is neglected, since it can hardly change the order of magnitude of the effect, (4-24) gives

$$\frac{\delta p_n(o)}{p_o(o)} \cong -\frac{1}{2}\,\frac{\tau^1_{1,1}}{T_o(o)} \qquad \text{for } \lambda = 1,\ s = 1,\ n = 1$$

$$\frac{\delta p^2_{2,2}(o)}{p_o(o)} \cong -2\,\frac{\tau^2_{2,2}}{T_o(o)} \qquad \text{for } \lambda = 2,\ s = 2,\ n = 2$$

$$\frac{\delta p^3_{3,3}(o)}{p_o(o)} \cong -\frac{5}{2}\,\frac{\tau^3_{3,3}}{T_o(o)} \qquad \text{for } \lambda = 3,\ s = 3,\ n = 3\ .$$

Although these relationships represent very crude approximations they show when compared with (4-19) that a temperature oscillation distributed evenly through the vertical extent of the atmosphere is several tens or a few hundred times as effective in inducing a tidal oscillation of surface pressure as is a surface temperature oscillation whose effects are spread upward by eddy conductivity. If the former had an amplitude near the equator of as much as 1/2°C, it would be competitive with the latter.

SECTION 5

SUMMARY AND CONCLUSIONS

This report has reviewed the elements of tidal theory as developed for the Earth's atmosphere and discussed the application of this theory to the Martian atmosphere. This theory leads to two ordinary differential equations, Laplace's tidal equation and the radial equation. Both contain a constant h, which arises in the separation of variables in an earlier partial differential equation.

In the study of tidal oscillations, one first specifies a period which is equal to or a submultiple of the solar day (in our atmosphere one is also interested in periods similarly related to the lunar day, but this has no application to Mars). For the specified period and a specified wave number, Laplace's tidal equation has a series of solutions in the form of Hough's functions, with each of which is associated a particular value of h, called an equivalent depth. Each of these solutions is spoken of as a mode of oscillation and describes the latitudinal behavior of the oscillation. Because the ratio of the length of the solar day to the length of the sidereal day is essentially the same on Mars as on Earth, a given mode of oscillation has essentially the same form in the two atmospheres. On the other hand, the equivalent depth corresponding to a given mode of oscillation is less on Mars than on Earth because of differences in radius and mass of the planets.

In the case of free oscillations (not gravitationally or thermally forced), the radial equation is soluble for only one or two values of h, which are spoken of as eigenvalues. The number and magnitude of the eigenvalue(s) depend on the average vertical temperature distribution. The Earth's atmosphere has only one such eigenvalue with a value of about 10 km. The Earth's atmosphere would have a second eigenvalue of about 8 km if the temperature near the stratopause were as high as was once thought likely. The temperature distribution in the Martian atmosphere is not well known. The temperature is believed to decrease upward, more rapidly near the surface than at higher levels. For a model atmosphere embodying these features, the Martian atmosphere also has one eigenvalue of about 20 km. This value does not depend very critically on the exact temperatures that are assumed. A second eigenvalue might arise if the temperature distribution at still higher levels were of a rather special character, but this possibility has not been explored.

The importance of the eigenvalues in tidal theory is as follows: if an excited mode of oscillation happens to have an equivalent depth whose value is very close to one of the eigenvalues, then that mode will be greatly amplified by resonance effects. A comparison of the equivalent depths on Mars for modes that might be excited by solar heating with the eigenvalue inferred for the Martian atmosphere indicates that no resonance magnification is to be expected.

Tidal oscillations on Mars might arise from the rather large diurnal temperature variation near the surface that is inferred from theory and observation. This possibility has been considered, and it appears highly improbable that the amplitude of the resulting diurnal surface-pressure oscillation (relative to the total surface pressure) exceeds the amplitude of Earth's semi-diurnal oscillation. If this conclusion is correct, tidal oscillations arising from this cause are not likely to play any significant role in the Martian general circulation.

Tidal oscillations might also arise from diurnal or semi-diurnal temperature oscillations caused by periodic radiative processes occurring through deep layers of the Martian atmosphere. In our present state of knowledge about the composition of the Martian atmosphere, one cannot be sure about the amplitude of such temperature oscillations, but they are probably too small to excite significant tidal motions. If the atmosphere should contain ozone in amounts greater than now suspected (or any other gas that absorbs significant amounts of solar radiation), then this tentative conclusion would have to be re-examined.

Thus, in the context of present inferences about temperature and composition of the Martian atmosphere, there is no reason to expect that tidal motions play an important role in the meteorology of Mars.

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